Steady Motion of a Sphere in a Rotating Fluid - A Numerical Study

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National Science Day 10th Research Scholars' Day 2020 IIT Bhubaneswar

1. Introduction

- Rotating fluid introduces many novel features into the motion of the fluid due to effect of rotation.
- Taylor (1921) observed experimentally that, when a sphere is allowed to move slowly through a fluid that is in a state of solid body rotation, a column of fluid is pushed ahead of the sphere like a solid mass having zero axial velocity relative to the moving body.
- This phenomenon is now known as the Taylor column and was predicted theoretically by Proudman (1916).
- A newly developed Higher order Compact scheme (HOCS) is used to capture the non-linear flow phenomena of rotating fluid accurately.

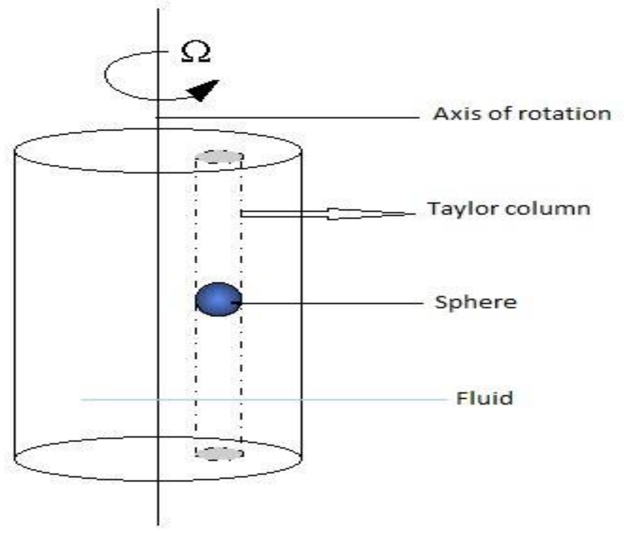

Figure 1: Taylor column experiment.

2. Objective

- To solve actual PDEs with higher accuracy and less cost of computation.
- To capture Taylor column, vortex jump phenomena and other aspects of experimental results of rotating fluid.

3. Why to use HOCS ?

- These are high accuracy finite difference approximations which is compact in nature.
- HOCS gives more accuracy even in coarser meshes.
- It consumes less CPU time and memory.
- Unconditionally stable.
- Easy boundary treatment (i.e. not having any ghost points).

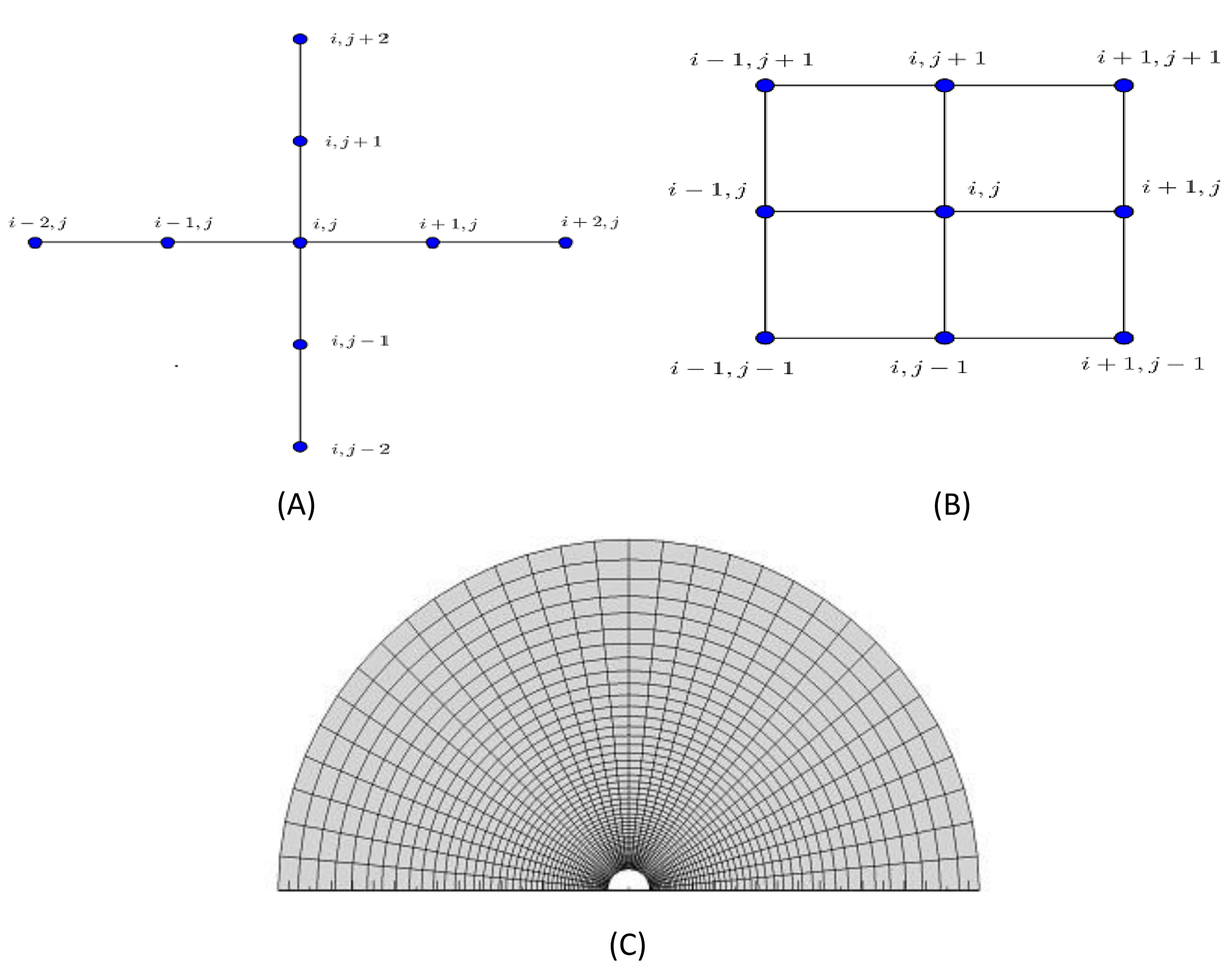

Figure 2: (A) Nine point non compact stencil; (B) Nine point compact stencil; (C) Domain used.

4. Formulation of the problem

Governing equations

$$\nabla \cdot \boldsymbol{q} = 0$$

$$\nabla \times \boldsymbol{q} = \omega$$

$$(\boldsymbol{q} \cdot \nabla)\, \boldsymbol{q} = \frac{1}{\rho}\nabla p + \nu \nabla^2 \boldsymbol{q}$$

where fluid velocity $\boldsymbol{q} = (q_r, q_\theta, q_\varphi)$ and q_r, q_θ, q_φ could be written in spherical form (r, θ, φ) in terms of stream function (ψ), vorticity (ω) and angular velocity (Ω) as

$$q_r = \frac{1}{r^2 \sin\theta}\frac{\partial \psi}{\partial \theta}, \qquad q_\theta = -\frac{1}{r\sin\theta}\frac{\partial\psi}{\partial r} \quad \text{and} \quad q_\varphi = \frac{\Omega}{r\sin\theta}.$$

- The dimensionless equations in spherical polar co-ordinates (r, θ, φ) with the transformation $r = e^{\xi}$ are

$$\frac{\partial^2\psi}{\partial\xi^2} + \frac{\partial^2\psi}{\partial\theta^2} - \frac{\partial\psi}{\partial\xi} - \cot\theta\frac{\partial\psi}{\partial\theta} = -e^{2\xi}\omega$$

$$\frac{\partial^2\omega}{\partial\xi^2} + \frac{\partial^2\omega}{\partial\theta^2} - \frac{\partial\omega}{\partial\xi} - \cot\theta\frac{\partial\omega}{\partial\theta} = \frac{Re}{e^{\xi}\sin\theta}\left\{\left(\frac{\partial\psi}{\partial\theta}\frac{\partial\omega}{\partial\xi} - \frac{\partial\psi}{\partial\xi}\frac{\partial\omega}{\partial\theta}\right)\right\}$$

$$+2\left(\cot\theta\frac{\partial\psi}{\partial\xi} - \frac{\partial\psi}{\partial\theta}\right)\omega - \frac{2T^2}{Re}\left(\cot\theta\frac{\partial\Omega}{\partial\xi} - \frac{\partial\Omega}{\partial\theta}\right)\Omega$$

$$\frac{\partial^2\Omega}{\partial\xi^2} + \frac{\partial^2\Omega}{\partial\theta^2} - \frac{\partial\Omega}{\partial\xi} - \cot\theta\frac{\partial\Omega}{\partial\theta} = \frac{Re}{e^{\xi}\sin\theta}\left\{\frac{\partial\psi}{\partial\theta}\frac{\partial\Omega}{\partial\xi} - \frac{\partial\psi}{\partial\xi}\frac{\partial\Omega}{\partial\theta}\right\}$$

where Re and T represents the Reynolds number and Taylor number respectively.

Boundary conditions

On the surface of the sphere ($\xi = 0$)

$$\psi = \frac{\partial\psi}{\partial\xi} = 0, \qquad \Omega = 0, \qquad \omega = -\frac{1}{\sin\theta}\frac{\partial^2\psi}{\partial\xi^2}$$

At large distances from the sphere ($\xi \to \infty$)

$$\psi = \frac{1}{2}e^{2\xi}\sin^2\theta, \qquad \Omega = e^{2\xi}\sin^2\theta, \qquad \omega = 0.$$

5. Validation

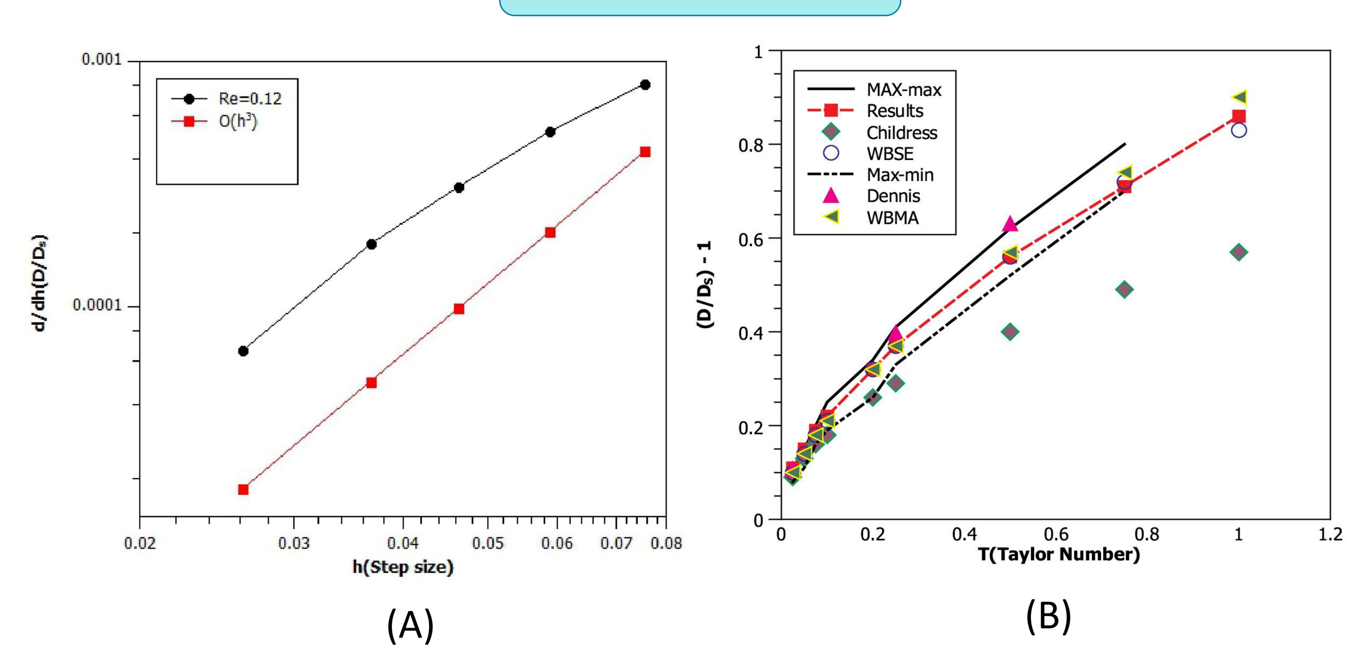

Figure 4: (A) Order of accuracy; (B) Comparison of drag values in presence of rotation (T) at Re = 0.12.

6. Results

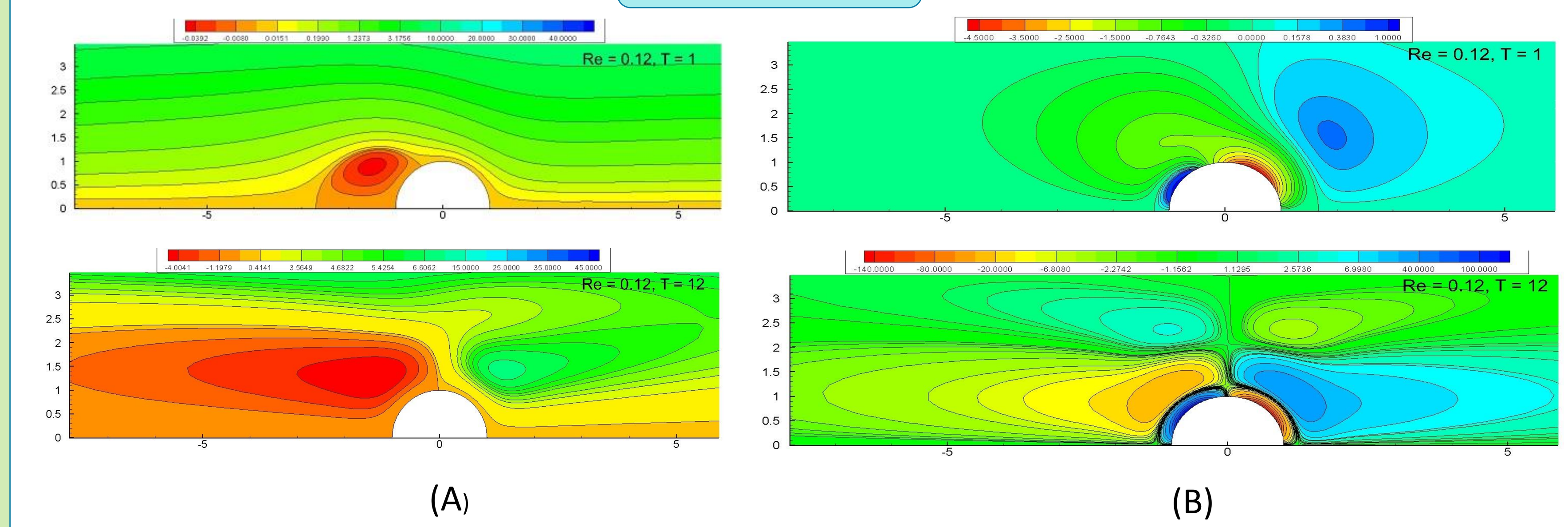

Figure 5: (A) Streamlines; (B) Vorticity contours at $Re = 0.12$ for different T

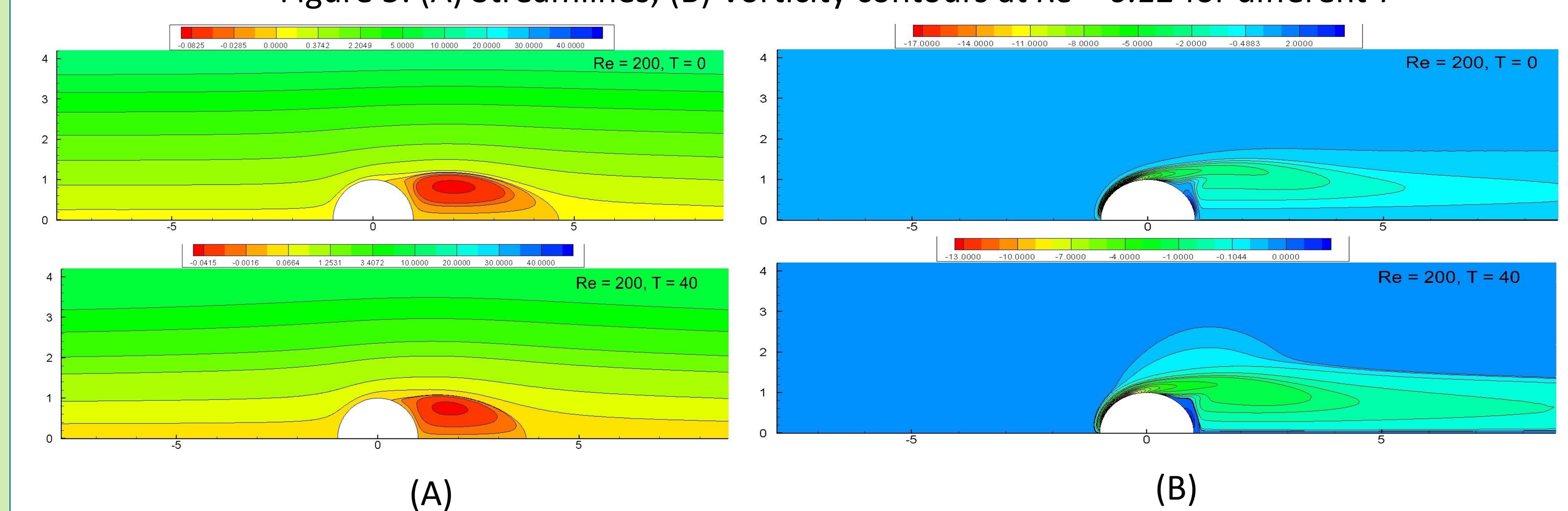

Figure 6: (A) Streamlines; (B) Vorticity contours at $Re = 200$ for different T.

7. Surface pressure

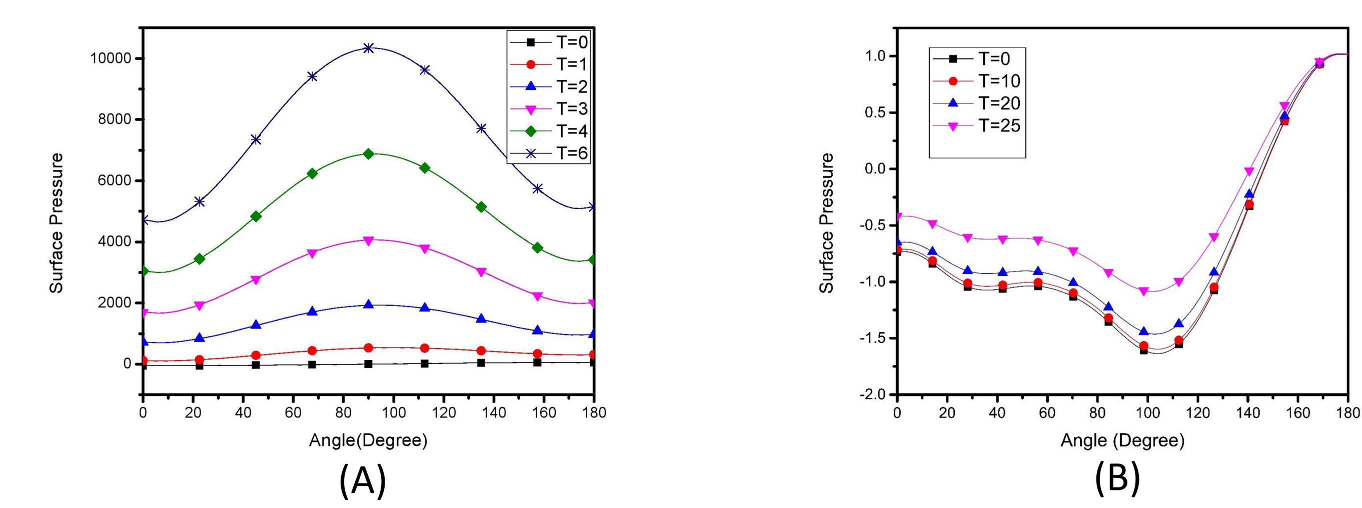

Figure 7: Pressure gradients on the surface of the sphere for (A) $Re=0.12$ (B) $Re=200$ for different T.

8. Conclusion

- It is found that for low values of Re drag values increases as rotation increases. Also, forward separation bubble is observed, whose volume increases with increase of Taylor number (T).
- This flow separation is called "Taylor Column" or "forward slug" in the theoretical studies of Tanzosh and Stone (1994} and the experimental studies of Maxworthy (1970).
- Downstream separation bubble which occurs due to the dominance of inertial forces around $Re = 25$ to steady high Re of 200 shrink in size with increase of the rotation of the fluid.

9. References

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